Modelling the Dynamics of Methamphetamine Abuse in the Western Cape

Asha Saidi Kalula

University of Stellenbosch
DST/NRF Centre of Excellence in Epidemiological Modelling and Analysis (SACEMA)
Supervisor : Dr. Farai Nyabadza
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University of Pretoria

- Illicit drugs are drugs whose use, possession or sale is illegal these drugs include cocaine, heroin and methamphetamine.
- Illicit drug use and related crime has imposed significant costs in different countries such as US, Australia, Japan, and South Africa.
- Methamphetamine is a powerful addictive stimulant that affects many areas of the central nervous system.
- Effects of MA includes increased energy and self confidence, heightened sense of sexuality, appetite suppression and weight loss

- Prolonged use of it is usually characterized by severe weight loss, higher risky of seizures, violent behaviour, confusion, impaired concentration and memory, and mood disturbances.

- Long term use increases the risky of contracting HIV and other infectious disease due to injection drug use and sexual risky behaviour.

- Motivation
  Dramatic increase in treatment demand for drugs and its implications to public health.

- To study the dynamics of methamphetamine abuse.
- To investigate the impact of behaviour change in methamphetamine abuse.
- To investigate the possibility of backward bifurcation in the developed model and its implications to public health.
- To investigate conditions under which methamphetamine abuse will persist or die out of the population.
- To project the number of methamphetamine users.
- To determine the incidence of methamphetamine abuse, that is estimating the number of individual who use methamphetamine abuse for the first time.

Table: Patients with methamphetamine as primary or secondary substance of abuse

Year	1996b	1997a	1997b	1998a	1998b	1999a
MA users	0	0	2	0	1	2
Year	1999b	2000a	2000b	2001a	2001b	2002a
MA users	6	10	12	14	17	21
Year	2002b	2003a	2003b	2004a	2004b	2005a
MA users	32	81	121	429	668	884
Year	2005b	2006a	2006b	2007a	2007b	2008a
MA users	952	1451	1413	1356	1209	

Year	2009a
MA users	1837

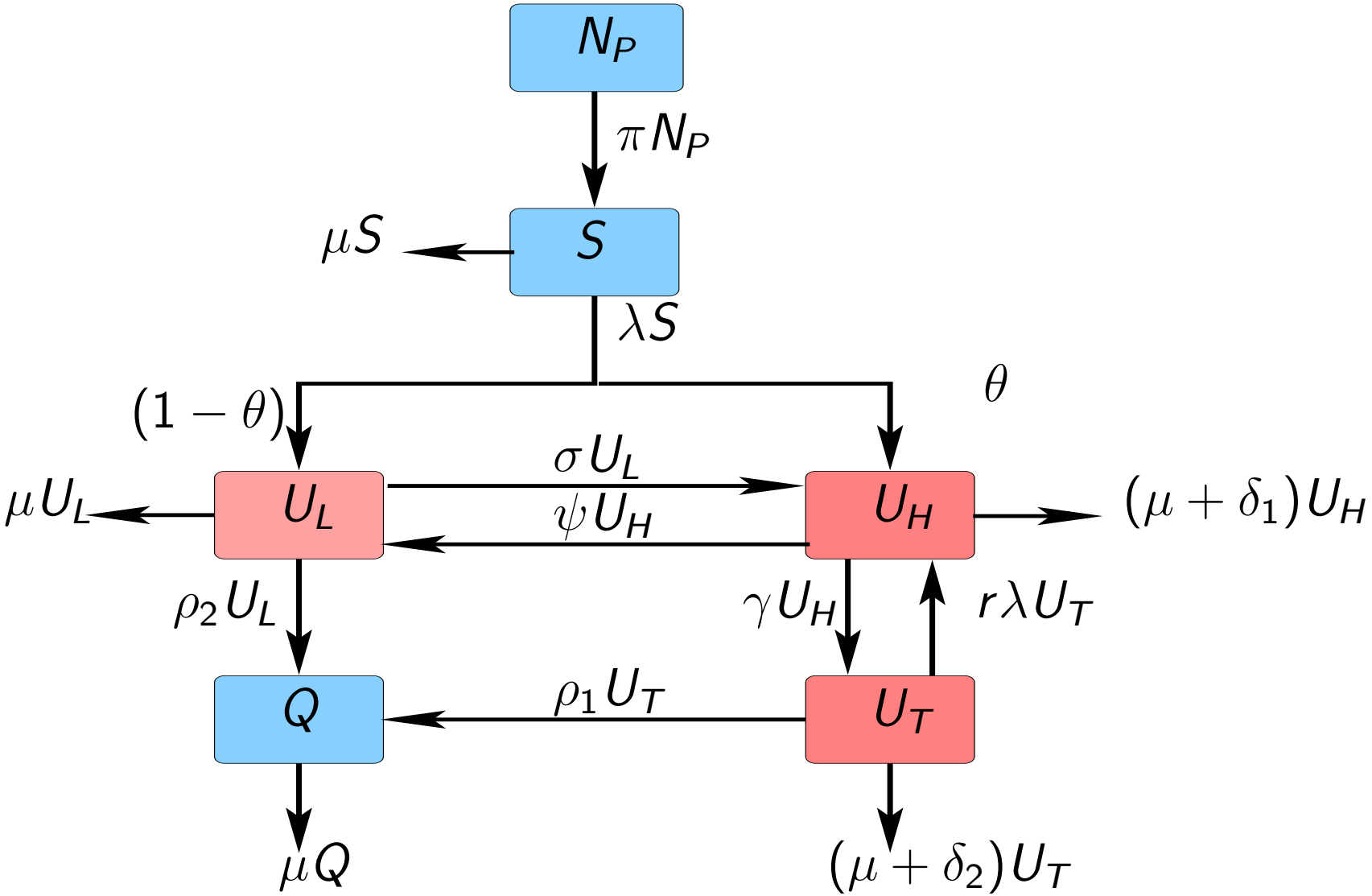

Figure: Flow diagram for the model

Parameter	Description
β	Transmission rate
σ	A rate of becoming a hard user
γ	Uptake rate into treatment
ψ	A rate of reversion to light drug use
r	Reinfection rate to being a hard drug user
ρ_1, ρ_2	Permanent recovery rate
δ_1, δ_2	Removal rates related to drug use
π	Recruitment rate
θ	Proportions of individuals who progress fast into U_H
μ	Natural mortality rate
η	Relative infectivity of U_H when compared to U_L
q	Measures of behavioural change
τ	A rate in which individuals starts drug on their own

- No removal or death related to drug use from light users compartment
- The removal or death related to drug use is different between hard drug users and drug users in treatment
- Hard drug users have low probability of generating new drug users than light drug users

$$\left.\begin{aligned}
\frac{dS}{dt} &= \pi N_P - (\mu + \lambda)S \\
\frac{dU_L}{dt} &= \lambda S(1-\theta) + \psi U_H - (\mu + \sigma + \rho_2)U_L \\
\frac{dU_H}{dt} &= \lambda S\theta + \sigma U_L + r\lambda U_T - (\mu + \gamma + \psi + \delta_1)U_H \\
\frac{dU_T}{dt} &= \gamma U_H - (\mu + \rho_1 + \delta_2 + r\lambda)U_T \\
\frac{dQ}{dt} &= \rho_2 U_L + \rho_1 U_T - \mu Q
\end{aligned}\right\} \quad (1)$$

where

$$\lambda = e^{-q(\delta_1 U_H + \delta_2 U_T)} \left[\tau + \beta \left(\frac{U_L + \eta U_H}{N_C} \right) \right]. \tag{2}$$

The mathematical analysis of system of equations (1) is not straight forward due to nonlinearities. We consider the case $q = 0$ and we leave the case $q > 0$ for numerical analysis for simplicity.

We express equilibria points in terms of λ as

$$S^* = \frac{\pi N_P}{\lambda^* + \mu}, \tag{3}$$

$$U_L^* = \pi\lambda^* N_P \left\{ \frac{r\lambda^* \left[\gamma\left(1-\theta\right) - \theta\psi\right] - \theta\psi b_3}{\omega} \right\} \tag{4}$$

$$-\pi\lambda^* N_P \left\{ \frac{b_2\left(1-\theta\right)\left(r\lambda^* + b_3\right)}{\omega} \right\},$$

$$U_H^* = \pi\lambda^* N_P \left\{ \frac{\left(r\lambda^* + b_3\right)\left[-\sigma\left(1-\theta\right) - \theta b_1\right]}{\omega} \right\}, \tag{5}$$

$$U_T^* = \pi\gamma\lambda^* N_P \frac{\left[-\sigma\left(1-\theta\right) - \theta b_1\right]}{\omega}, \tag{6}$$

$$
\begin{aligned}
Q^* &= -\pi\lambda^* N_P \left\{ \frac{\gamma\left[\sigma\left(1-\theta\right)\theta b_1\right]\rho_1 + r\lambda^*\left[-\gamma\left(1-\theta\right)+\theta\psi\right]\rho_2}{\mu\omega} \right\} \\
&\quad -\pi\lambda^* N_P \left\{ \frac{r\lambda^*[\theta\psi b_3 + b_2\left(1-\theta\right)\left(r\lambda^*+b_3\right)]\rho_2}{\mu\omega} \right\},
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
b_1 &= (\mu+\sigma+\rho_2) \\
b_2 &= \gamma+\psi+\mu+\delta_1 \\
b_3 &= (\rho_1+\mu+\delta_2) \\
q_1 &= \frac{\sigma\psi}{b_1 b_2} \\
\omega &= (\lambda^*+\mu)\left[\sigma\psi\left(r\lambda^*+b_3\right)+b_1\left\{r\gamma\lambda^* - b_2\left(r\lambda^*+b_3\right)\right\}\right].
\end{aligned}
$$

Substituting U_L^*, U_H^*, U_T^* and $N_C^* = S^* + U_L^* + U_H^* + U_T^* + Q^*$ into (2), we obtain

$$G(\lambda) = A\lambda^3 + B\lambda^2 + C\lambda + D = 0, \tag{8}$$

with

$$\begin{aligned}
A &= \pi r\mu N_P(1-\theta)(\gamma - b_2) - \pi r\mu\sigma N_P(1-\theta) - \pi r\theta\mu\psi N_P \\
&\quad -\pi r\theta\mu b_1 N_P + \pi r\rho_2 N_P(1-\theta)(\gamma - b_2) - \pi r\theta\psi\rho_2 N_P \\
B &= -\pi r\beta\gamma\mu N_P(1-\theta) - \pi\gamma\mu\sigma N_P(1-\theta) - \pi r\gamma\mu\tau N_P(1-\theta) \\
&\quad +\pi r\mu\sigma\tau N_P(1-\theta) + \pi r\beta\theta\mu\psi N_P + \pi r\mu\sigma\psi N_P + \pi r\theta\mu\tau\psi N_P \\
&\quad +\pi r\gamma\mu b_1 N_P - \pi\gamma\theta\mu b_1 N_P + \pi r\theta\mu\tau b_1 N_P - \pi r\mu b_1 b_2 N_P \\
&\quad +\pi r\beta\mu b_2 N_P(1-\theta) + \pi r\mu\tau b_2 N_P(1-\theta) - \pi\theta\mu b_1 b_3 N_P \\
&\quad -\pi\mu b_2 b_3 N_P(1-\theta) - \pi\mu\sigma b_3 N_P(1-\theta) - \pi\theta\mu\psi b_3 N_P \\
&\quad +\pi r\beta\mu\sigma\eta N_P(1-\theta) + \pi r\beta\theta\mu b_1\eta N_P - \pi\gamma\sigma\rho_1 N_P(1-\theta) \\
&\quad -\pi\gamma\theta b_1\rho_1 N_P - \pi r\gamma\tau\rho_2 N_P(1-\theta) + \pi r\theta\tau\psi\rho_2 N_P
\end{aligned}$$

$$
\begin{aligned}
& +\pi r\tau b_2\rho_2 N_P\left(1-\theta\right)-\pi\theta\psi b_3\rho_2 N_P-\pi b_2 b_3 N_P\left(1-\theta\right)\rho_2 \\
C \;=\; & \pi\gamma\mu\sigma\tau N_P\left(1-\theta\right)-\pi r\mu\sigma\tau\psi N_P-\pi r\gamma\mu\tau b_1 N_P \\
& +\pi r\mu\tau b_1 b_2 N_P+\pi\mu\sigma\tau b_3 N_P\left(1-\theta\right)+\pi\beta\theta\mu\psi b_3 N_P \\
& +\pi\theta\mu\tau\psi b_3 N_P+\pi\theta\mu\tau b_1 b_3 N_P+\pi\beta\mu b_2 b_3 N_P\left(1-\theta\right) \\
& -\pi\mu b_1 b_2 b_3 N_P+\pi\beta\mu\sigma b_3\eta N_P\left(1-\theta\right)+\pi\beta\theta\mu b_1 b_3\eta N_P \\
& +\pi\gamma\theta\tau b_1\rho_1 N_P+\pi\theta\tau\psi b_3\rho_2 N_P+\pi\tau b_2 b_3\rho_2 N_P\left(1-\theta\right) \\
& +\pi\gamma\theta\mu\tau b_1 N_P+\pi\mu\sigma\psi b_3 N_P+\pi\mu\tau b_2 b_3 N_P\left(1-\theta\right) \\
& +\pi\gamma\sigma\tau\rho_1 N_P\left(1-\theta\right) \\
D \;=\; & \pi\mu\tau b_1 b_2 b_3 N_P\left(1-q_1\right).
\end{aligned}
$$

If $\tau = 0$, D becomes zero and the polynomial becomes

$$G(\lambda) = \lambda(A\lambda^2 + B_1\lambda + C_1) = 0, \tag{9}$$

where

$$\begin{aligned}
B_1 &= \pi r\beta\theta\mu\psi N_P - \pi r\beta\gamma\mu N_P(1-\theta) - \pi\gamma\mu\sigma N_P(1-\theta) \\
&\quad + \pi r\mu\sigma\psi N_P + \pi r\beta\theta\mu b_1 N_P\eta - \pi r\mu b_1 b_2 N_P \\
&\quad + \pi r\gamma\mu b_1 N_P - \pi\theta\mu\psi b_3 N_P - \pi b_2 b_3 N_P\rho_2(1-\theta) \\
&\quad - \pi\gamma\theta\mu b_1 N_P + \pi r\beta\mu b_2 N_P(1-\theta) - \pi\mu\sigma b_3 N_P(1-\theta) \\
&\quad - \pi\theta\mu b_1 b_3 N_P - \pi\mu b_2 b_3 N_P(1-\theta) + \pi r\beta\mu\sigma N_P\eta(1-\theta) \\
&\quad - \pi\gamma\sigma N_P\rho_1(1-\theta) - \pi\gamma\theta b_1 N_P\rho_1 - \pi\theta\psi b_3 N_P\rho_2, \\
C_1 &= \pi\beta\mu b_1 b_2 b_3 N_P\left[\frac{\theta\psi}{b_1 b_2} + \frac{(1-\theta)}{b_1} + \eta\left\{\frac{\sigma(1-\theta)}{b_1 b_2} + \frac{\theta}{b_2}\right\}\right] \\
&\quad - \pi\mu b_1 b_2 b_3 N_P(1-q_1).
\end{aligned}$$

- Drug free equilibrium
  $E_0 = (S^*, U_L^*, U_H^*, U_T^*, Q^*) = (\frac{\pi N_P}{\mu}, 0, 0, 0, 0)$.
- Methamphetamine epidemic threshold R_0

$$R_0 = \frac{\beta}{(1-q_1)}\left[\frac{\theta\psi}{b_1 b_2} + \frac{(1-\theta)}{b_1} + \eta\{\frac{\sigma(1-\theta)}{b_1 b_2} + \frac{\theta}{b_2}\}\right].$$

The interpretation of R_0 is as follows:

- $\frac{1}{b_1}$ refers to the duration methamphetamine users spends in light drug use stage,
- $\frac{1}{b_2}$ the duration methamphetamine users spends in hard drug use stage,
- $\frac{\theta\psi}{b_1 b_2} + \frac{(1-\theta)}{b_1}$ is the contribution of light drug users,
- $\eta\left\{\frac{\sigma(1-\theta)}{b_1 b_2} + \frac{\theta}{b_2}\right\}$ is the contribution of hard drug users.

If $\tau > 0$ we have the polynomial (8) with $A < 0$, $D > 0$, B either positive or negative and C expressed as

$$C = \pi\mu b_1 b_2 b_3 N_P(1-q_1)[R(\tau)-1]$$

where

$$\begin{aligned} R(\tau) \;=\; & \frac{(\beta+\tau)}{(1-q_1)}\left[\frac{\theta\psi}{b_1 b_2}+\frac{(1-\theta)}{b_1}\right]+\frac{(\beta\eta+\tau)}{(1-q_1)}\left[\frac{\sigma(1-\theta)}{b_1 b_2}+\frac{\theta}{b_2}\right] \\ & +\frac{\gamma\tau}{(1-q_1)}\left[\frac{\sigma(1-\theta)}{b_1 b_2 b_3}+\frac{\theta}{b_2 b_3}\right]+\frac{\gamma\tau\rho_1}{\mu(1-q_1)}\left[\frac{\sigma(1-\theta)}{b_1 b_2 b_3}\right] \\ & +\frac{\gamma\tau\rho_1}{\mu(1-q_1)}\left[\frac{\theta}{b_2 b_3}\right]+\frac{\tau\rho_2}{\mu(1-q_1)}\left[\frac{\theta\psi}{b_1 b_2}+\frac{(1-\theta)}{b_1}\right] \\ & +\frac{r\tau}{(1-q_1)}\left[\frac{(\mu+\delta_1)(\mu+\sigma+\rho_2)+\psi(\mu+\rho_2)}{b_1 b_2 b_3}\right]. \end{aligned}$$

- Note that when $\tau = 0$, $R(\tau) = R_0$.

Therefore, when $q = 0$ and $\tau > 0$ we have the following results

Theorem

- *a unique drug persistent equilibrium if* $R_\tau > 1$,
- *one drug persistent equilibria if* $B < 0$ *and* $R_\tau < 1$,
- *at most three drug persistent equilibria if* $B > 0$ *and* $R_\tau < 1$,
- *no drug persistent equilibrium otherwise.*

Remark

In the presence of innovators, drug persistence is always guaranteed. We were unable to explicitly determine the equilibria, but the analysis has been helpful in showing that drug use will always persist.

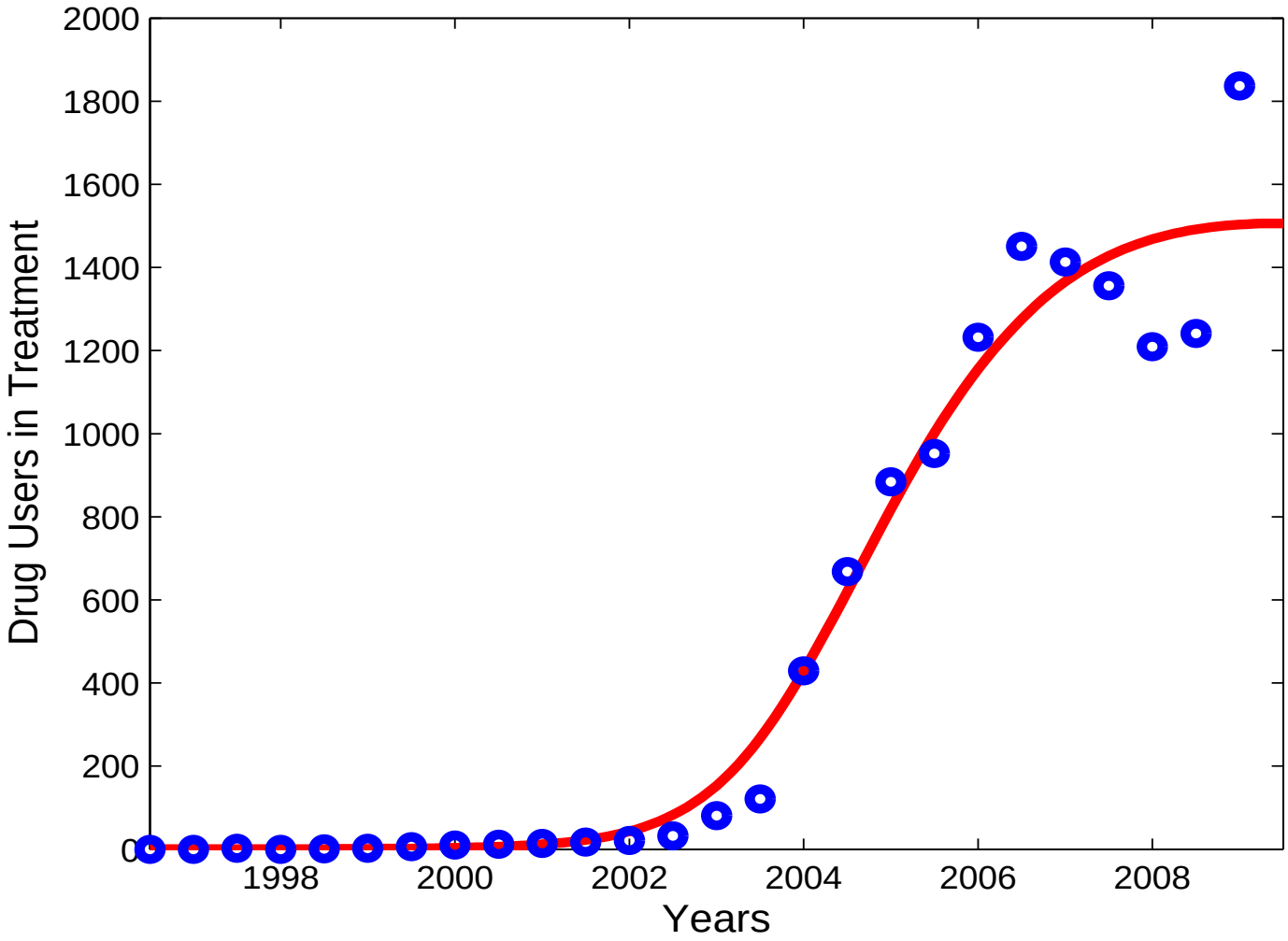

Figure: Shows the change in the population of individuals under treatment //

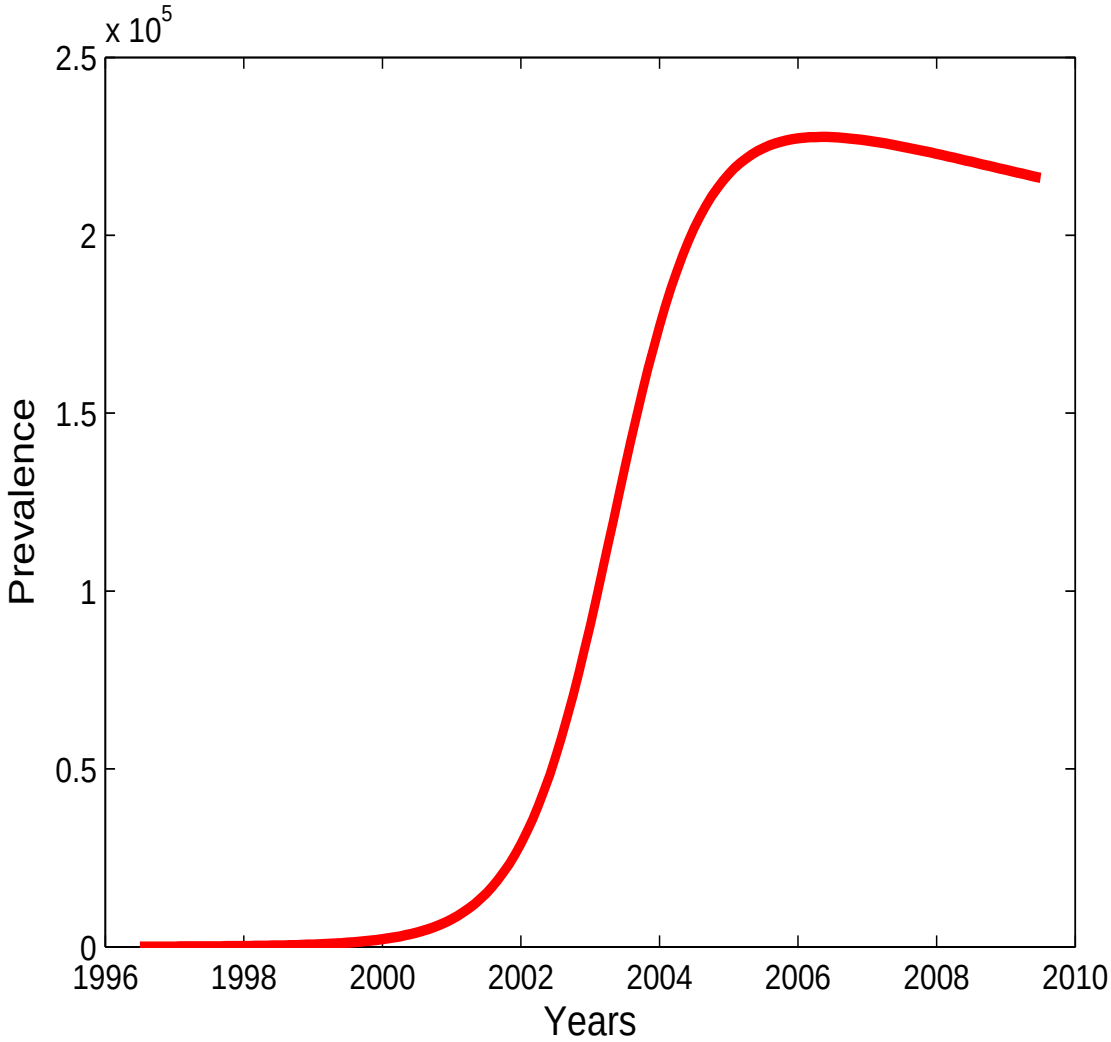

Figure: Shows the change on prevalence over time.

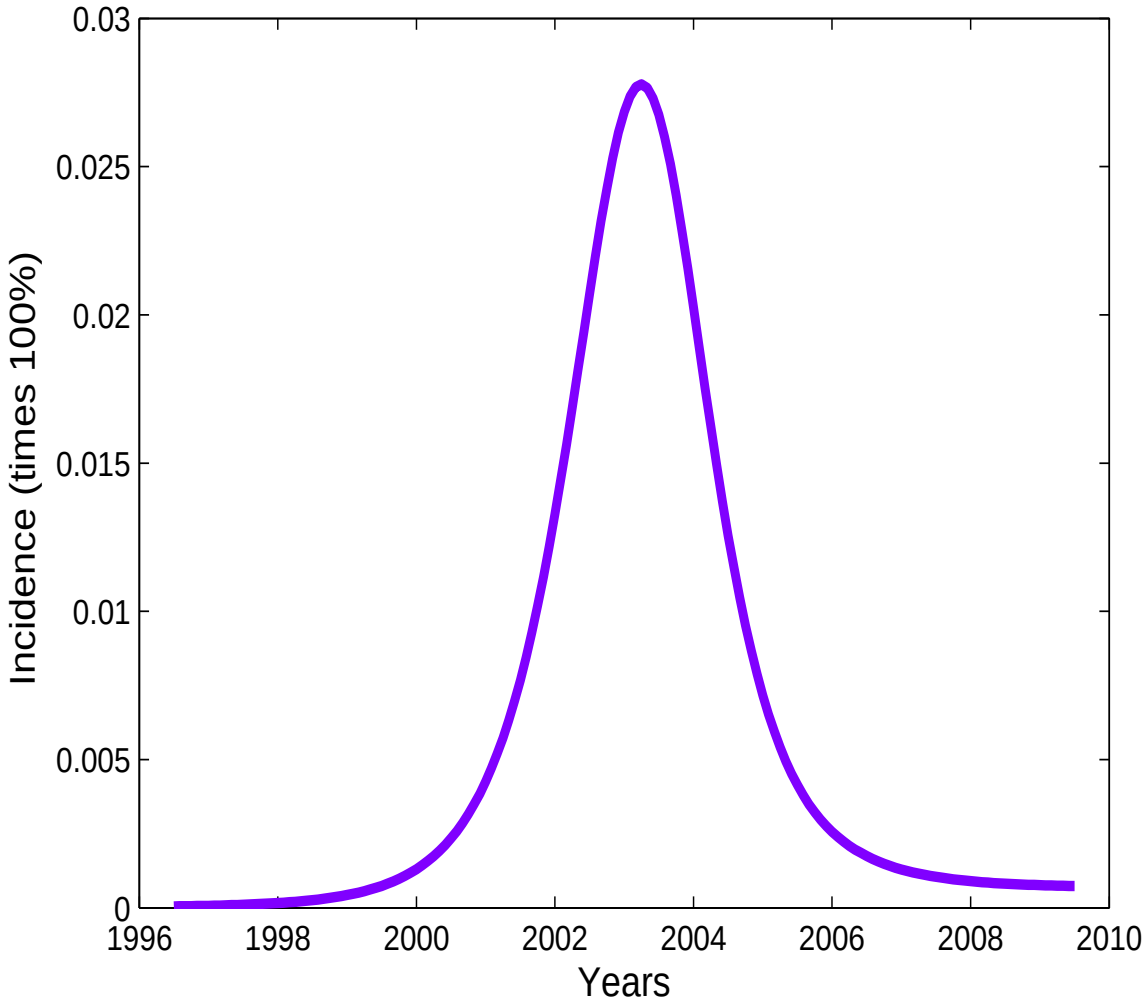

Figure: Shows the change on incidence with time.

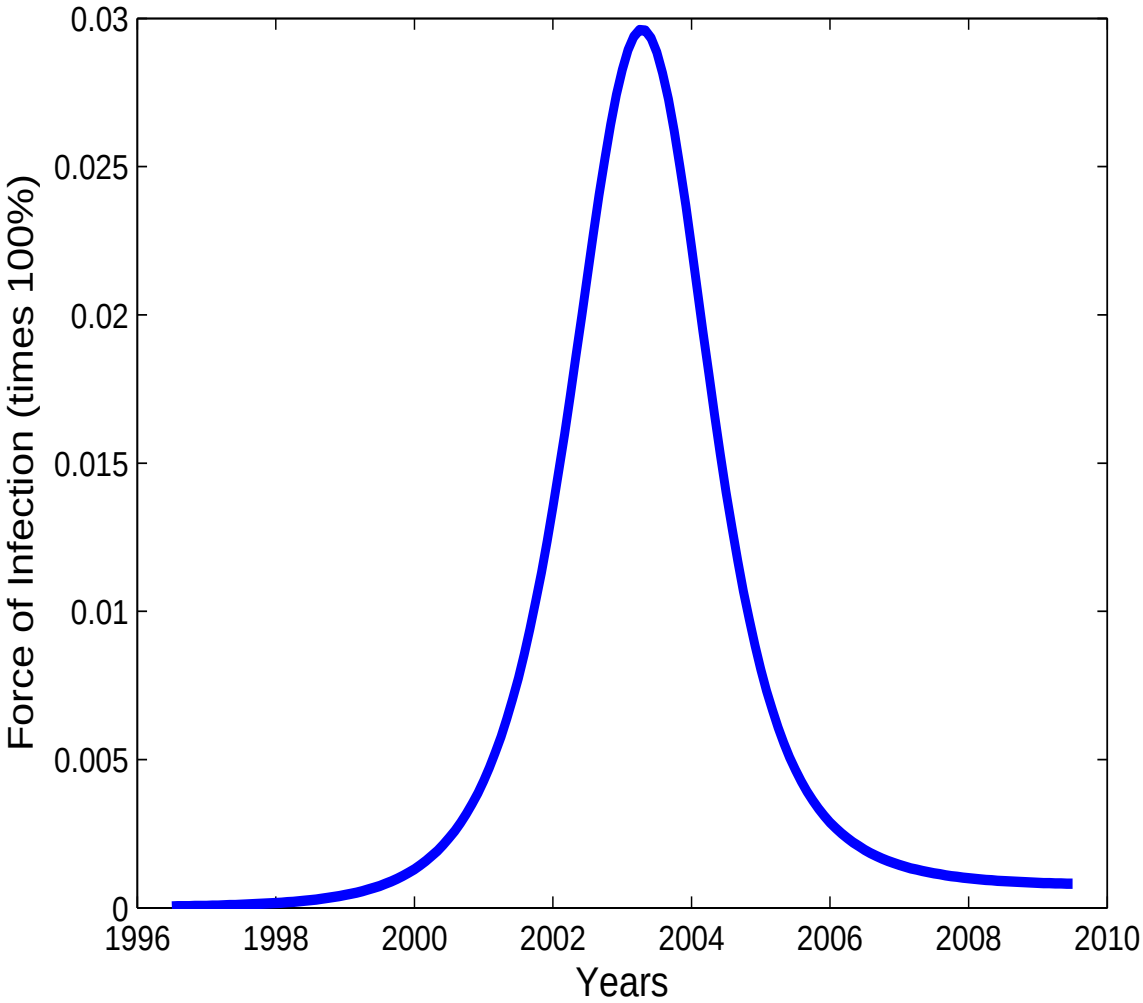

Figure: Shows the change on force of infection over time.

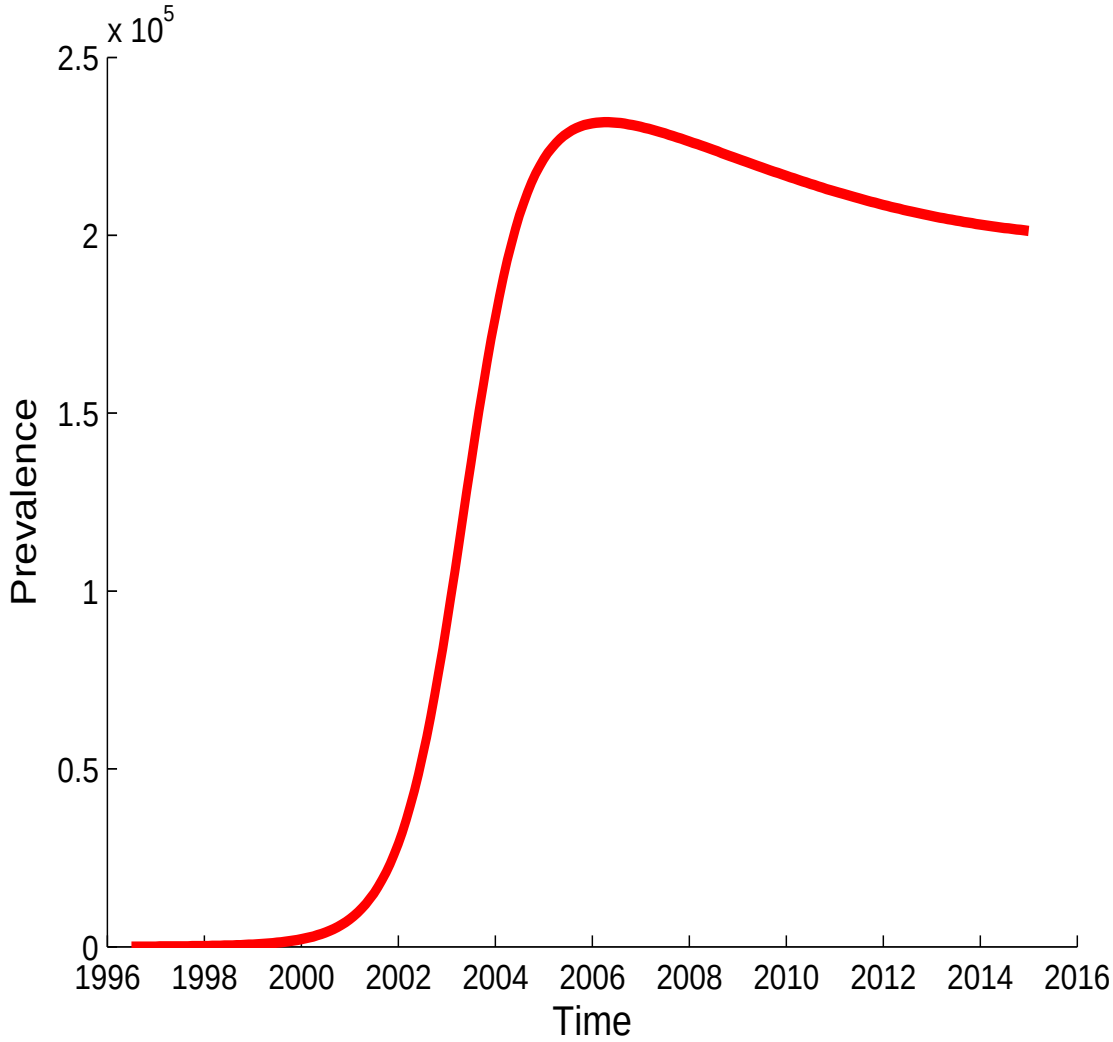

Figure: Shows the change on prevalence over time.

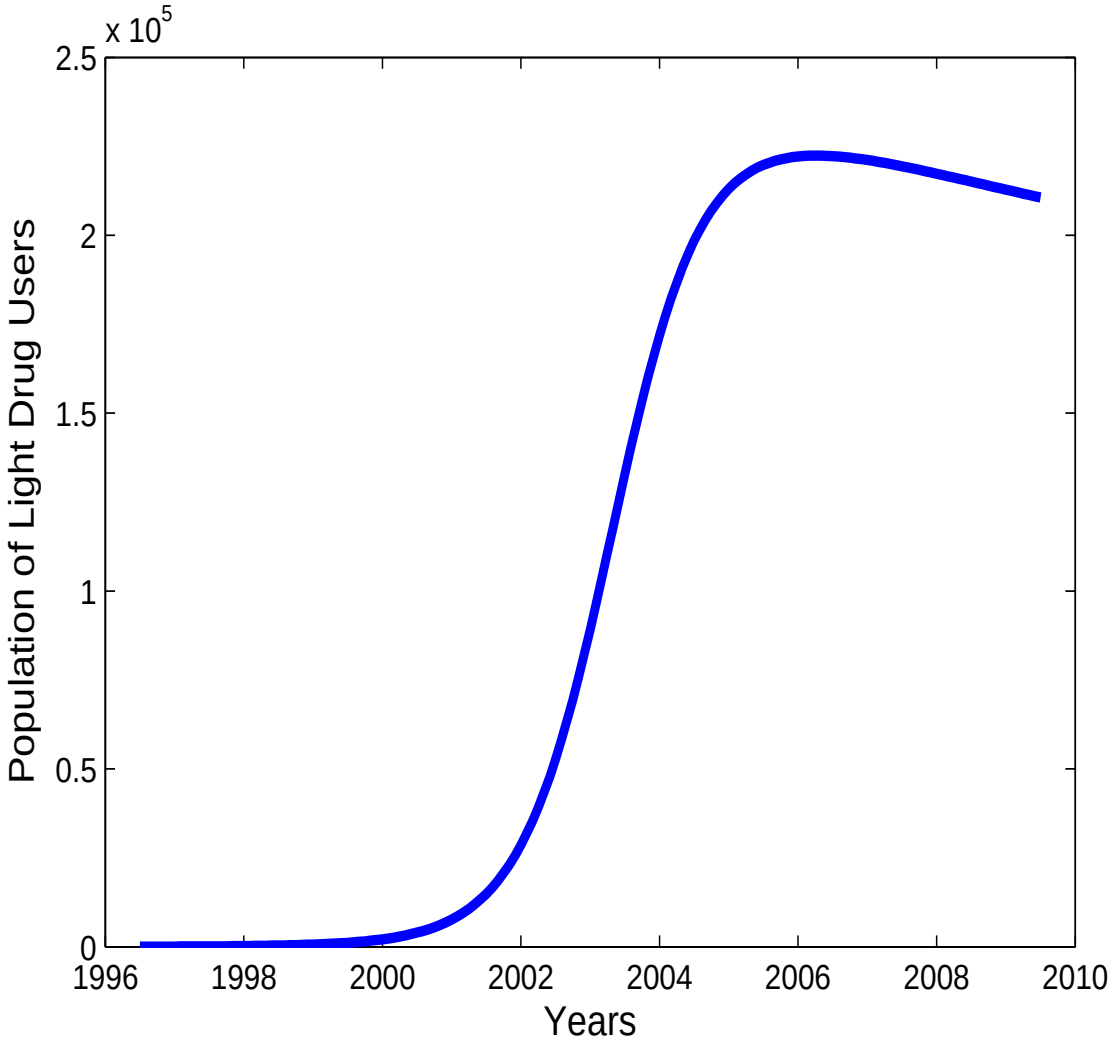

Figure: Population of light drug users.

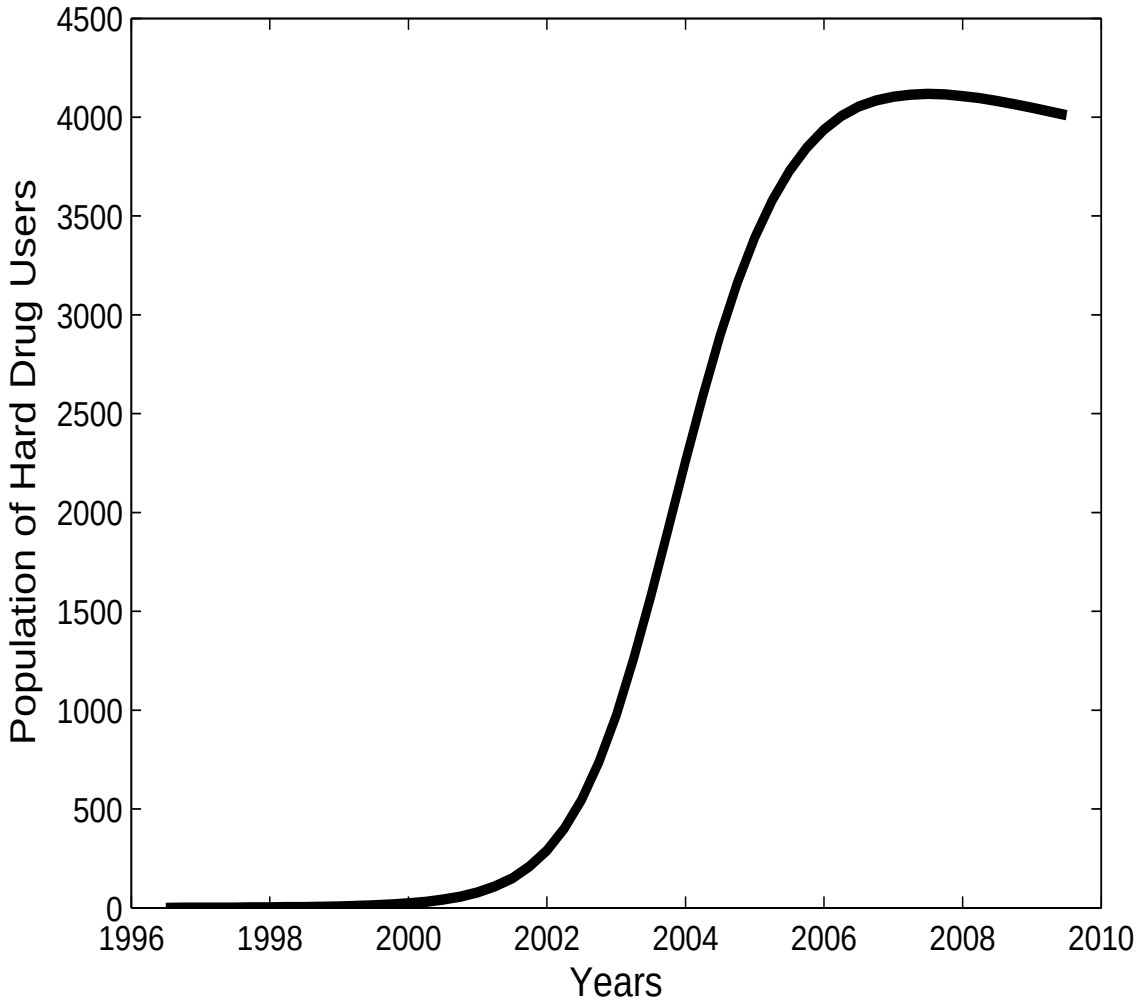

Figure: Polpulation of hard drug users.

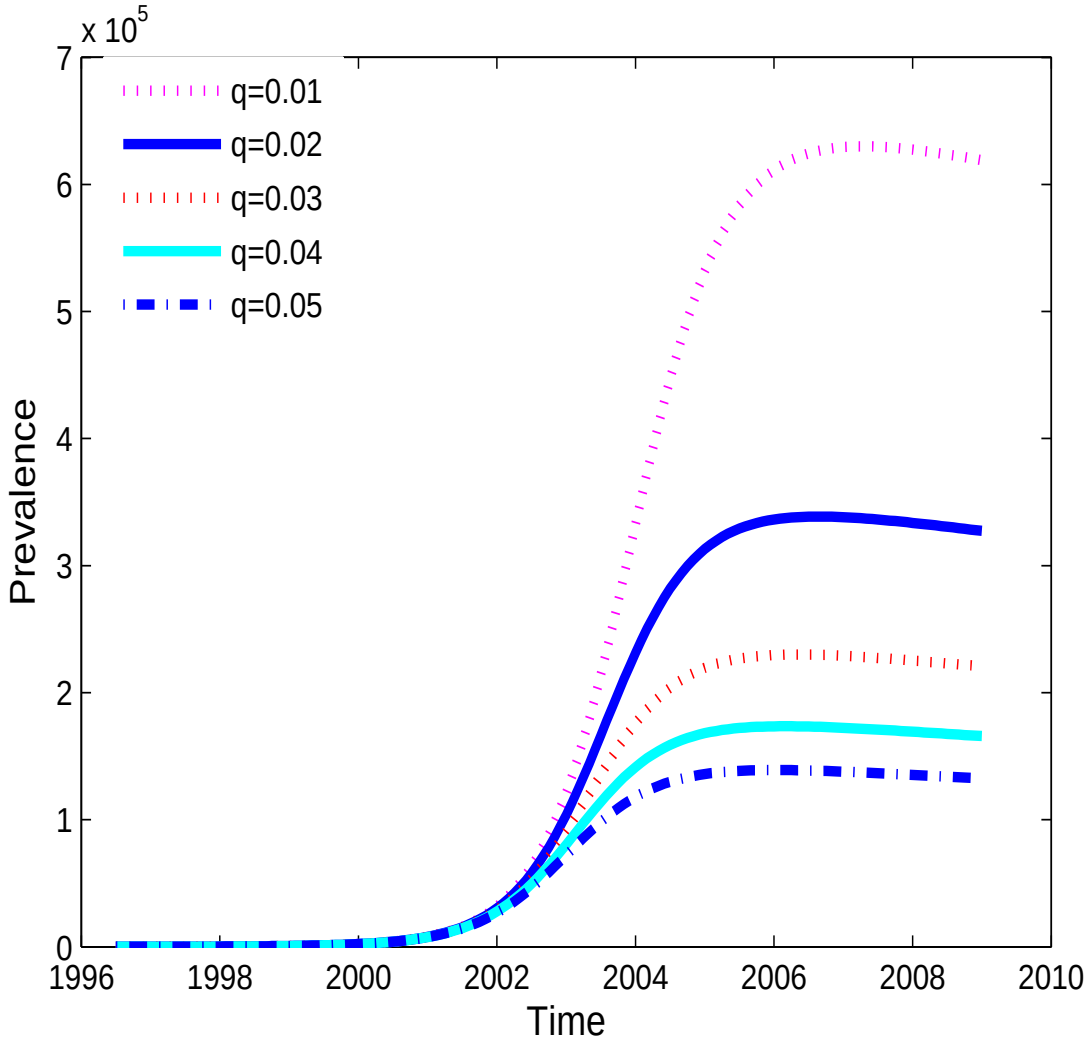

Figure: Shows the impact of behaviour change on prevalence.

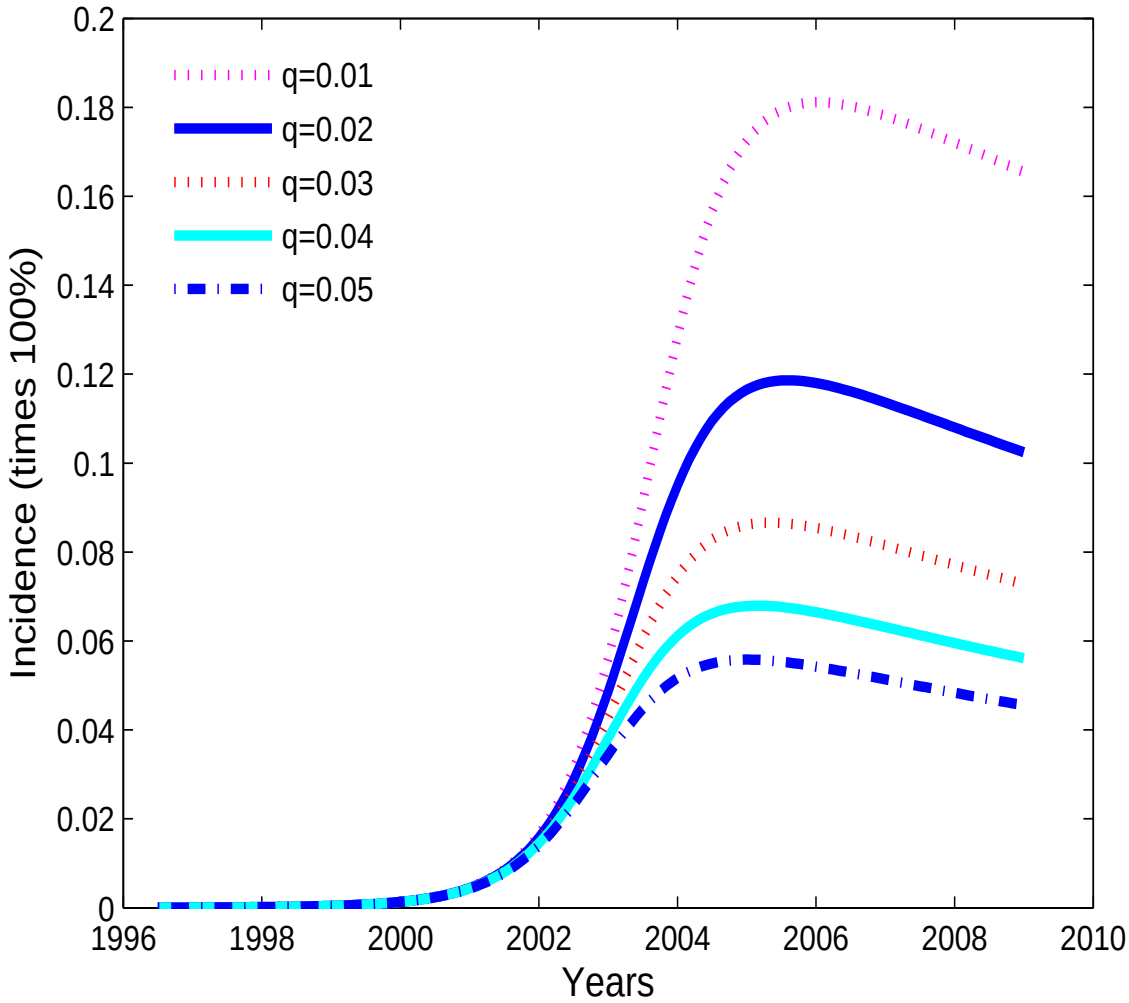

Figure: Shows the impact of behaviour change on incidence.

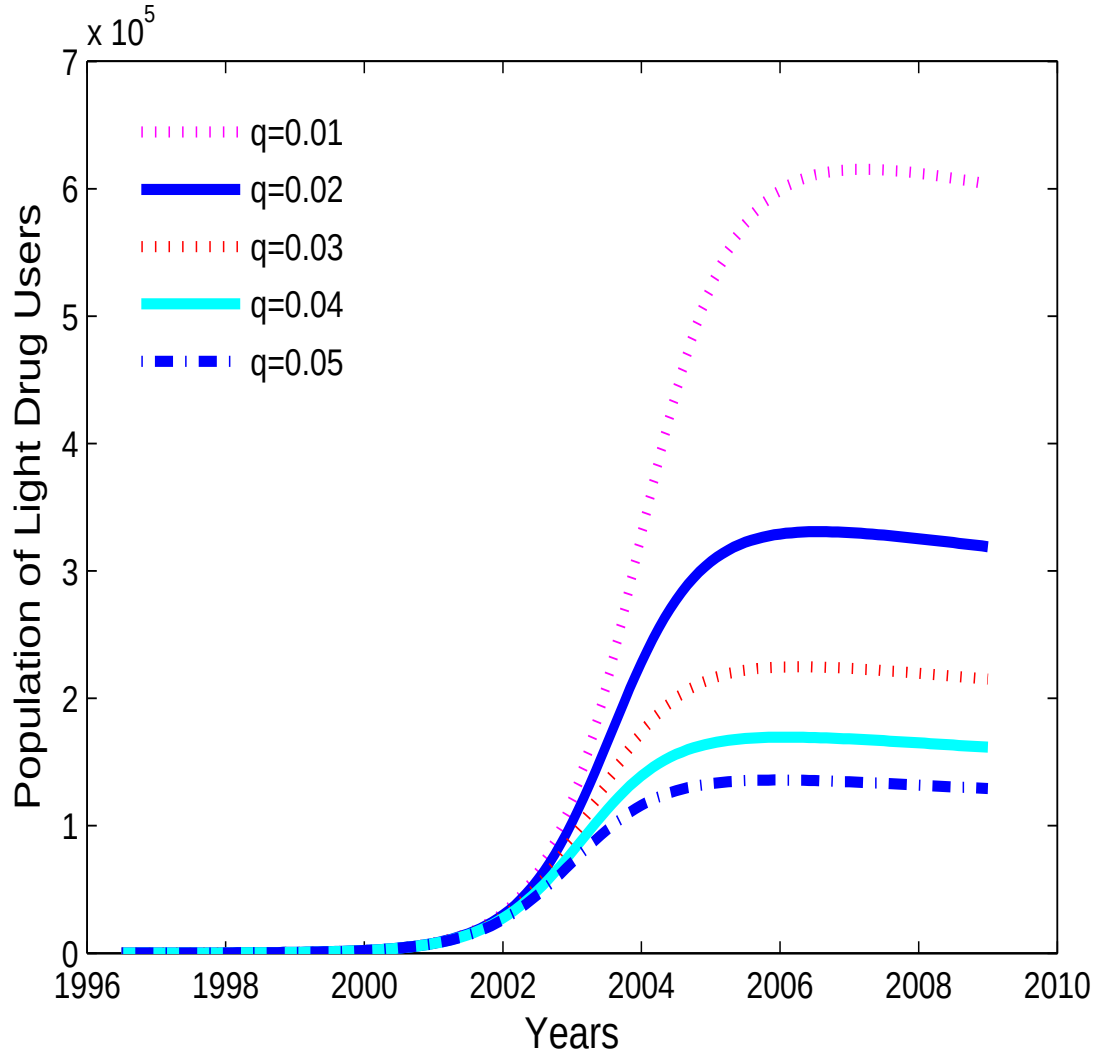

Figure: Shows the impact of behaviour change on light drug users.

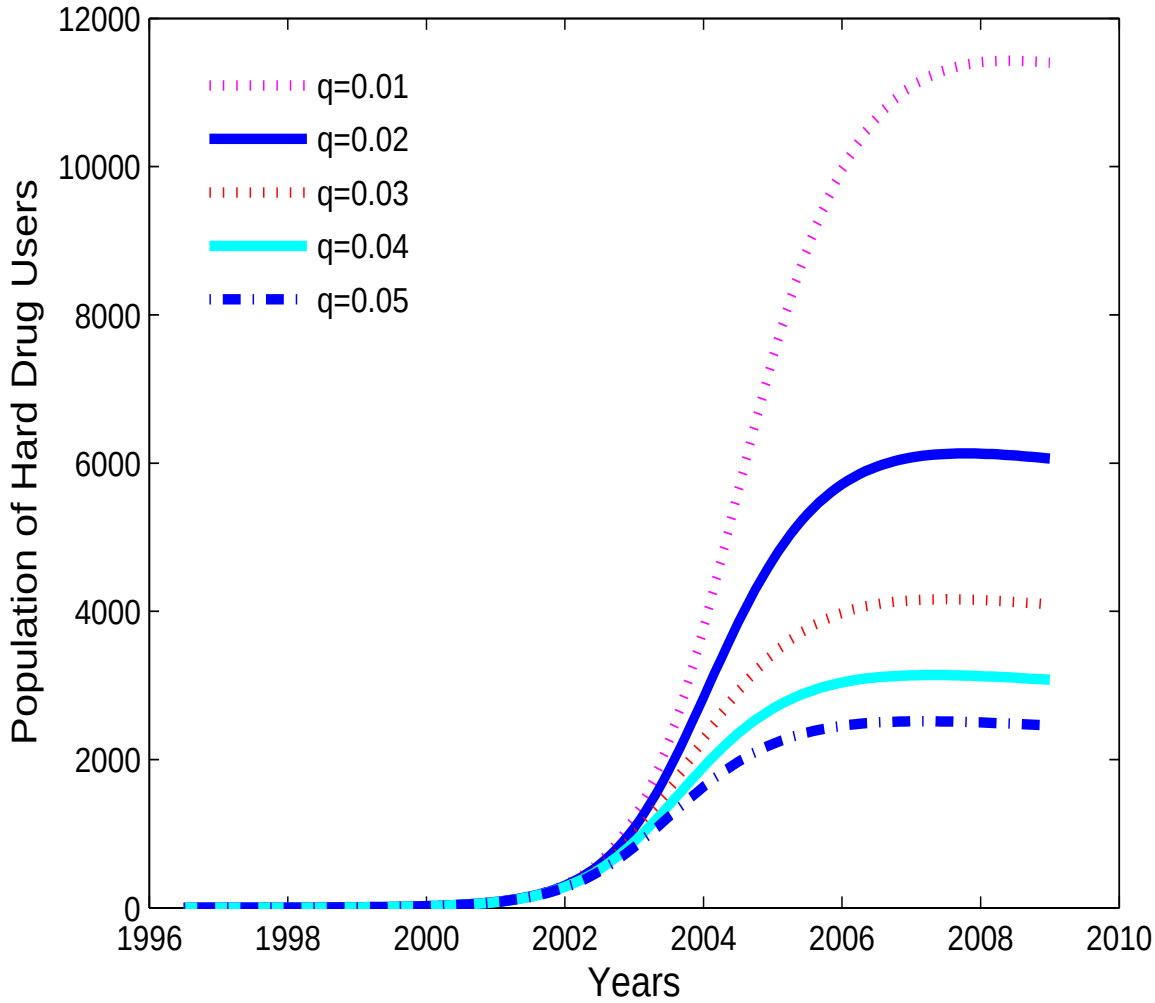

Figure: Shows the impact of behaviour change on hard drug users.

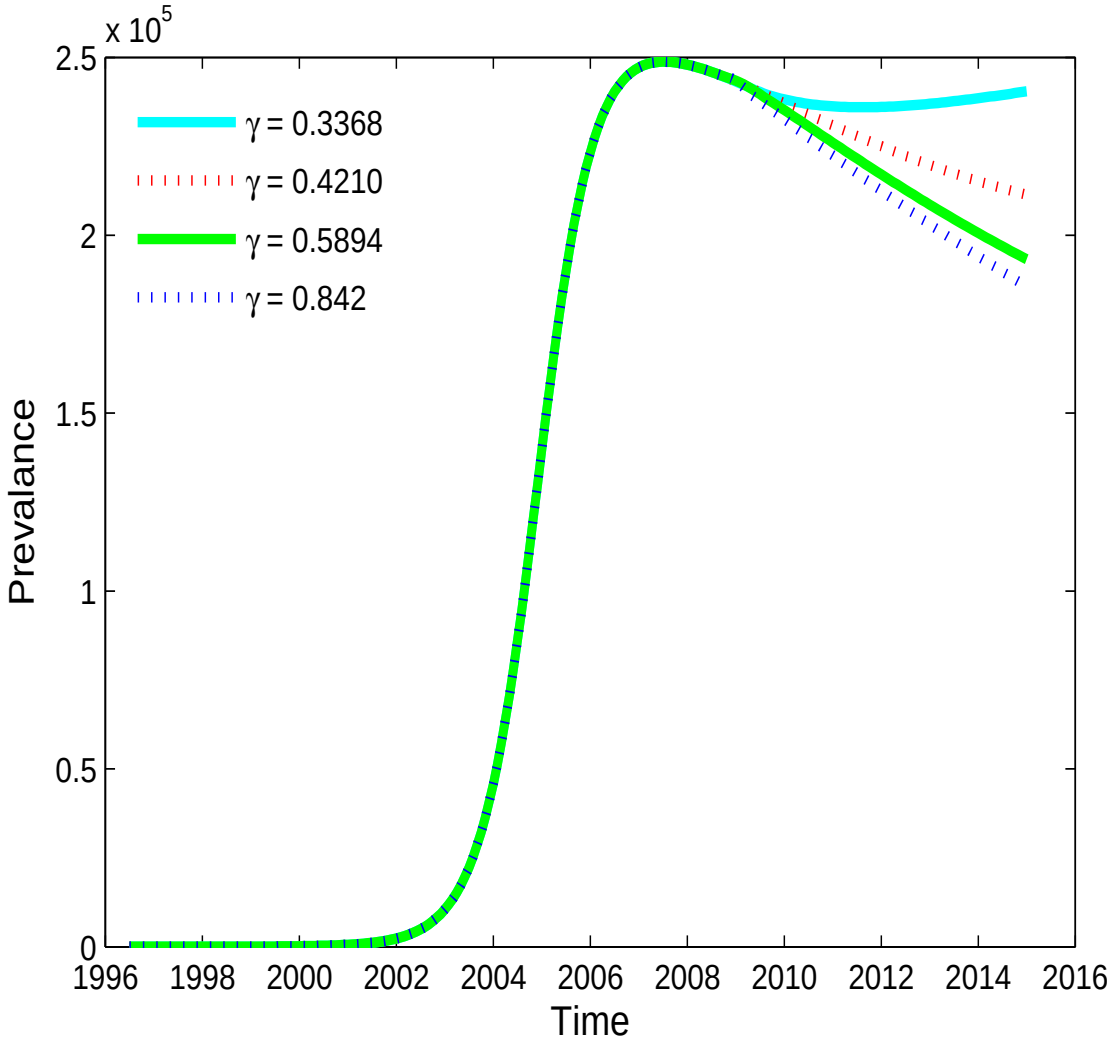

Figure: Impacts of uptake rate into treatment (γ) on prevalence.

- There more light drug users than hard and drug users in treatment.
- There more than 227.600 drug users.
- The number of drug users as well as prevalence can be reduced by increasing uptake rete through increasing treatment centers.
- The number of drug users can also be reduced by having intervations which focuses on light drug use class.
- It can also be reduced by having prevention or intervention programs which focuses on changing behaviour.

- Doris . A. Behren, Jonathan P. Caulkins, Gernot Tragler, Josef L. Haunschmied and Gustav Feichtinger. A dynamic model of drug initiation: implications for treatment and drug control (1999).
- Emma White and Catherine Cosmiskey. Heroin epidemics, treatment and ODE modeling (2006)
- Leickness .C. Simbayi, Seth .C. Kalichman, Demetria Cain, Charsey Cherry, Namvo Henda and Allanise Cloete. Methamphetamine use and sexual risks for HIV infection in Cape Town, South Africa (2006).
- Farai Nyabadza and Senelani D Hove-Musekwa. From heroin epidemics to methamphetamine epidemics: Modeling substance abuse in a South African province (2010).